Discrete Optimization

TECHNISCHE UNIVERSITÄT DARMSTADT

Resilient and Efficient Layout of Water Distribution Networks

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Joint work with Andreas Schmitt
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within the Collaborative Research Center 805

SFB 805
Control of Uncertainty in Load-Carrying Structures in Mechanical Engineering

Overview

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Topics of this talk:

- ▷ water distribution networks for tall buildings
- ▷ global solution of mixed-integer nonlinear programs
- ▷ energy efficient control of pumps
- ▷ topology/layout optimization
- ▷ robust optimization (resilience, interdiction)
- ▷ $\rightsquigarrow$ min max min structure

Goals of this talk:

- ▷ show how particular structure can help
- ▷ demonstrate methods for resilience

Literature – Selection

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Water network optimization:

- D'Ambrosio et al (2015) [overview]
- Mala-Jetmarova, Sultanova, and Savic (2017) [models]
- Kolb and Lang (2012) [PDE]
- Geißler et al. (2012) [control with integer variables]

Topology optimization:

- De Corte A, Sörensen K (2013) [overview]
- Bragalli et al. (2012) [diameter optimization]

Robustness:

- Robinius et al. (2018) [tree shaped robust networks]
- Meng et al. (2018) [resilience]

Outline

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1 Water Distribution Networks – Model
2 Branch and Bound Solving Scheme
3 Resilience
4 Concluding Remarks

High-Rise Water Supply Systems

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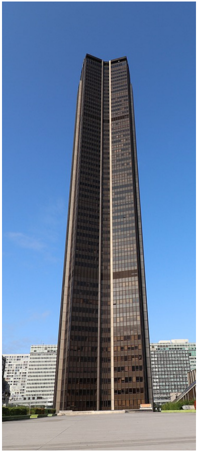
For water supply in high buildings:

- ▷ Need pumps to overcome gravity.
- ▷ Corresponding energy costs are significant.



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High-Rise Water Supply Systems

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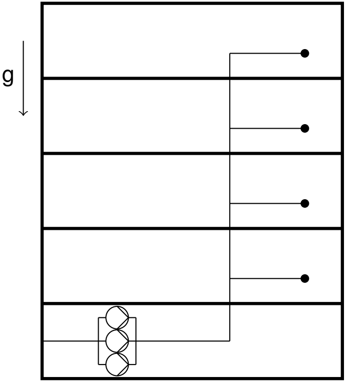
State of the art:

- ▷ Place pumps in basement and connect floors by one pipe strand.
- ▷ Energy inefficient: would need less pressure for lower levels.



High-Rise Water Supply Systems

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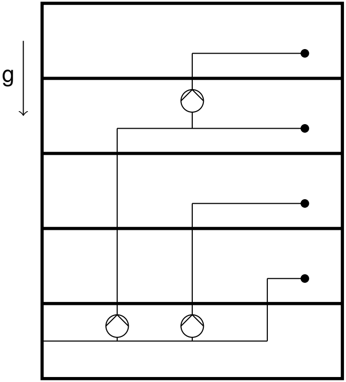
Decentralized approach:

Optimize costs of

- ▷ interconnection of pressure zones,
- ▷ placement of pumps, and
- ▷ operating speed of pumps

to supply building with water.



Mixed-Integer Nonlinear Program (MINLP)

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- Steady state setting
- Continuous variables for physical quantities:
  - pressure head h,
  - volume flow q,
  - power p, and
  - normalized rotating speed ω.
- Binary variables for the following decisions:
  - Which pipe should be selected?
  - Where to place a pump of a given pump type?
- Constraints (non-convex):
  - nonlinear pump characteristics,
  - hydraulic resistance laws,
  - flow conditions in the pipe network, and
  - binary decisions for components (on/off).
- Objective:
  - Minimize combination of operating and investment costs.

Pipe Connections

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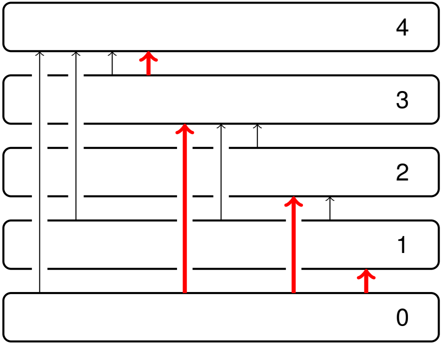
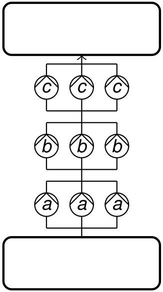
- Base graph $G = (V, A)$
  - $V = \{0, \dots, N\}$
  - $A = \{(u, v) \in V \times V \ : \ u < v\}$
- $x_a = 1$ if connection a is used
- Feasible connections
  - form a spanning tree rooted in 0
  - $$\sum_{a \in \delta^-(v)} x_a = 1, \quad v \in V$$



Pump Placement

- Set of different pump types $\mathcal{C}$
- On each connection different types in series
- Up to M of each type in parallel
- $y^m_{a,i} = 1$ if pump type i is built m times on a
- $\sum_{m=1}^{M} y^m_{a,i} \leq x_a, \quad a \in A, i \in \mathcal{C}$



$\mathcal{C} = \{a, b, c\}$

$M = 3$

Flow and Pressure

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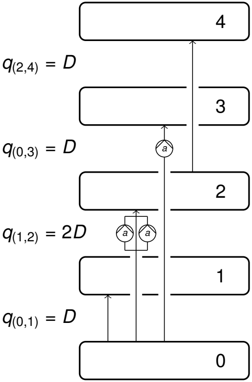
- Volume flow demand of D at each pressure zone
- q_a = flow along connection a
- h_v = pressure difference of zone v to inlet
- $\Delta h_{a,i}$ = pressure increase of type i pumps on a
- Flow balance
- Input and minimal pressure
  $h_0 = 0,\ h_v \geq H_v^{\min},\ v \in V \setminus \{0\}$
- Friction on pipe R (Darcy-Weisbach)
  $R_a(q) = \lambda \frac{1}{d_a^5} \frac{8}{\pi^2} \frac{q^2}{g} L_a$
- Pressure distribution for $a = (u, v) \in A$:
  $$x_a = 1 \ \Rightarrow\ h_v \leq h_u + \sum_{i \in \mathcal{C}} \Delta h_{a,i} - L_a - R_a(q_a)$$



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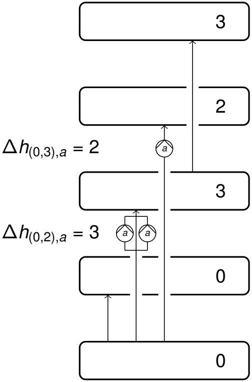

Pump Characteristic Diagrams

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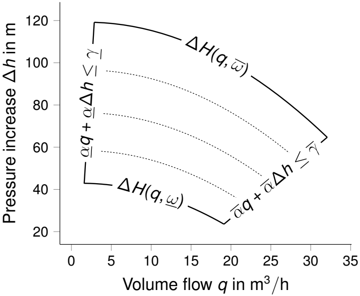

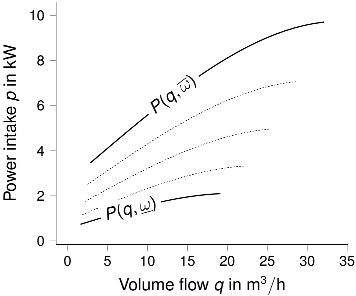

- $\omega_{a,i}$ relative operating speed of type i pumps on a
- ω, q, Δh and p related by characteristic diagrams
- polynomial approximations of power intake and pressure increase:
  - $\Delta H_i(q, \omega) = \alpha_i^H \, q^2 + \beta_i^H \, q\,\omega + \gamma_i^H \, \omega^2$
  - $P_i(q, \omega) = \alpha_i^P \, q^3 + \beta_i^P \, q^2\,\omega + \gamma_i^P \, q\,\omega + \delta_i^P \, \omega^3$

Mixed-Integer Nonlinear Program

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$$
\begin{aligned}
\min\ & \sum_{a\in A} C^{\text{pi}}_a x_a + \sum_{a\in A}\sum_{i\in\mathcal{C}}\sum_{m=1}^{M} C^{\text{pu}}_i\, m\, y^m_{a,i} + C^{\text{en}} \sum_{a\in A}\sum_{i\in\mathcal{C}} p_{a,i} \\
\text{s.t.}\ & \sum_{a\in\delta^-(v)} q_a - \sum_{a\in\delta^+(v)} q_a = D, && v\in V, && 0\le q_a \le N\,D\,x_a, && a\in A,\\
& \Delta h_{a,i} = \sum_{m=1}^{M} \Delta H_i\Big(\frac{q_a}{m},\omega_{a,i}\Big)\, y^m_{a,i}, && a\in A, i\in\mathcal{C}, && \sum_{m=1}^{M} y^m_{a,i}\le 1, && a\in A, i\in\mathcal{C},\\
& p_{a,i} = \sum_{m=1}^{M} m\, P_i\Big(\frac{q_a}{m},\omega_{a,i}\Big)\, y^m_{a,i}, && a\in A, i\in\mathcal{C}, && \sum_{a\in\delta^-(v)} x_a = 1, && v\in V,\\
& \Big(\overline{\alpha}_i \frac{q_a}{m} + \overline{\beta}_i\,\Delta h_{a,i} - \overline{\gamma}_i\Big) y^m_{a,i} \le 0, && a\in A, i\in\mathcal{C}, m\in[M], && h_v \ge \underline{H}, && v\in V\setminus\{0\},\\
& \Big(h_v - h_u - \sum_{i\in\mathcal{C}}\Delta h_{a,i} + L_a + R_a(q_a)\Big)x_a = 0, && a=(u,v)\in A, && h_0 = H_0,\\
& x\in\{0,1\}^A,\ y\in\{0,1\}^{A\times\mathcal{C}\times[M]},\ q\in\mathbb{R}^A_+,\ \omega\in[\underline{\omega},1]^{A\times\mathcal{C}},\ \Delta h\in\mathbb{R}^{A\times\mathcal{C}}_+,\ p\in\mathbb{R}^{A\times\mathcal{C}}_+,\ h\in\mathbb{R}^V_+.
\end{aligned}
$$

Example of Optimal Solution

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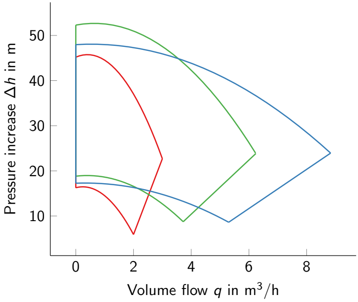
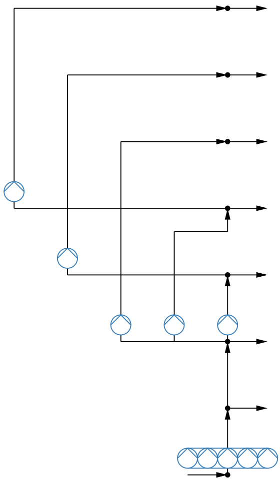
- Hotel, 100 m tall, 7 pressure zones
- Total flow demand 28 m^3/h
- Operating time 21000 h
- 3 pump types, placeable up to 5 times in parallel





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3 Resilience
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Branch and Bound Algorithm

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init $U = \infty$ and $\mathcal{T} = \{T\}$ with $V_T = \{0, 1\}$ and $A_T = \{(0, 1)\}$
while $\mathcal{T} \neq \emptyset$ **do**
    choose and remove a partial tree T from $\mathcal{T}$
    solve relaxation for T and denote its optimal value O
    if $O < U$ *(else fathom node)*
        if T *spans* G
            solve exact problem for T and denote its optimal value O^*
            update $U = \min\{U, O^*\}$
        else
            form new partial trees from T and add to $\mathcal{T}$ (branching)
return U;

- Exploit tree property: Pipe topology determines volume flow.
- Enumerate partial trees of G using a Branch and Bound scheme.
- Subproblems: optimal placement and operation of pumps for given tree.

Optimal Pump Placement and Operation
Hidden convexity for fixed flow

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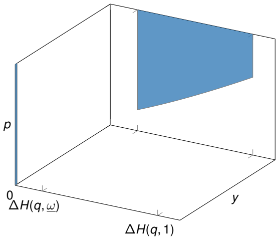
Pump Description

$$\{(y, p, \Delta h) \in \{0,1\} \times \mathbb{R}^2_+ \,:\, p \geq \tilde{P}_q(\Delta h)\, y,\; \Delta H(q, \underline{\omega})\, y \leq \Delta h \leq \Delta H(q, 1)\, y\}$$

- Consider fixed flow q.
- $\tilde{P}_q(\Delta h)$ formed by elimination of ω from $\Delta H(q, \omega)$ and plugging into $P(q, \omega)$.
- Observation: $\tilde{P}_q(\Delta h)$ is convex for the given bounds
- Verified for particular pumps by solving a MINLP
- $\Rightarrow$ perspective cuts ([Frangioni, Gentile 2006])



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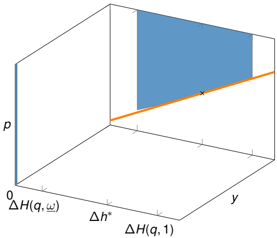

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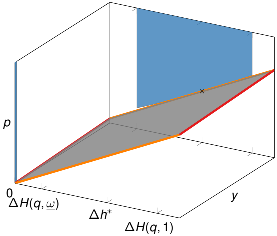
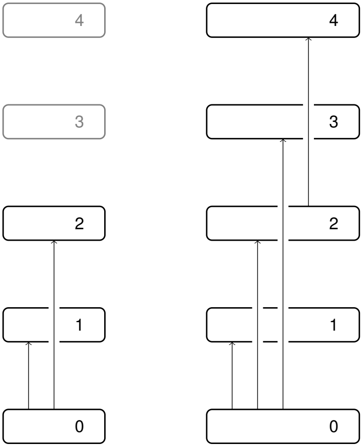
- ▷ Verified for particular pumps by solving a MINLP
- ▷ $\Rightarrow$ perspective cuts ([Frangioni, Gentile 2006])
- ▷ $p \geq \tilde{P}'_q(\Delta h^*)\Delta h + \left(\tilde{P}_q(\Delta h^*) - \tilde{P}'_q(\Delta h^*)\Delta h\right) y.$
- ▷ Validity: $y = 0 \rightarrow \Delta h = 0 \rightarrow p \geq 0$; $y = 1 \rightarrow p \geq \tilde{P}_q(\Delta h)\, y$.



Relaxation

For partial tree T:

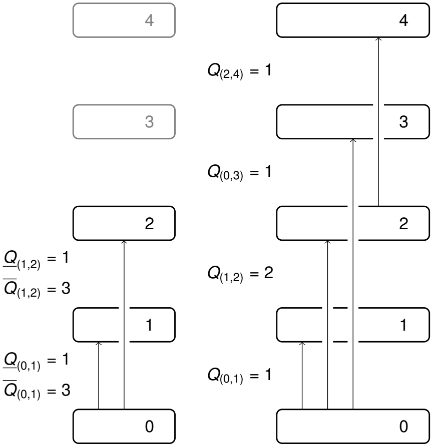
- Fixed path $\mathcal{P}_v^T$ from 0 to each $v \in V_T$
- supply only zones in V_T



Relaxation

For partial tree T:

- Fixed path $\mathcal{P}_v^T$ from 0 to each $v \in V_T$
- supply only zones in V_T
- bound flow on connections in A_T



Relaxation

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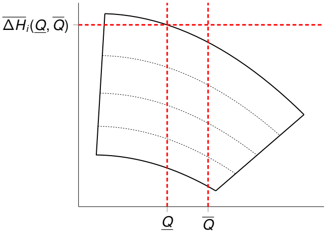
For partial tree T:

- Fixed path $\mathcal{P}_v^T$ from 0 to each $v \in V_T$
- supply only zones in V_T
- bound flow on connections in A_T
- relax characteristic diagram using best-case values



$$\overline{\Delta H}_i(\underline{Q}, \overline{Q}) = \max \quad \Delta h$$
$$\text{s.t.} \quad (\Delta h, q, \omega) \text{ is feasible for pump } i,$$
$$q \in [\underline{Q}, \overline{Q}]$$

$$\underline{P}_i(\underline{Q}, \overline{Q}) = \min \quad P_i(q, \omega)$$
$$\text{s.t.} \quad (\Delta h, q, \omega) \text{ is feasible for pump } i,$$
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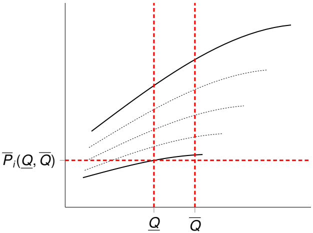

Relaxation Model

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Relaxation model for partial tree T

$$\min \quad \sum_{a \in A_T} C_a^{\text{pi}} x_a + \sum_{a \in A_T} \sum_{i \in \mathcal{C}} \sum_{m=1}^{M} m \left(C_i^{\text{pu}} + C^{\text{en}} \, \underline{P}_i\left(\tfrac{\underline{Q_a}}{m}, \tfrac{\overline{Q_a}}{m}\right) \right) y_{a,i}^m$$

$$\text{s.t.} \quad \sum_{a \in \mathcal{P}_v^T} \sum_{i \in \mathcal{C}} \sum_{m=1}^{M} \overline{\Delta H}_i\left(\tfrac{\underline{Q_a}}{m}, \tfrac{\overline{Q_a}}{m}\right) y_{a,i}^m \geq H^{\text{min}}, \qquad v \in V_T \setminus \{0\},$$

$$\sum_{m=1}^{M} y_{a,i}^m \leq 1, \qquad a \in A_T, i \in \mathcal{C},$$

$$y \in \{0,1\}^{A_T \times \mathcal{C} \times [M]}.$$

Computational Results

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Comparison of MINLP and adapted branch and bound scheme:

# zones	MINLP			B-and-B		
	gap	time	solved	time	time relax	solved
4	0.00	111.27	36	2.20	0.29	36
5	3.90	1383.68	31	11.24	1.81	36
6	50.51	5929.19	10	58.02	11.45	36
7	142.31	7200.00	0	412.62	92.23	36
8	239.19	7200.00	0	3315.81	786.77	36

- SCIP 6.0.2, CPLEX 12.8.0, 2h time limit
- Gap: arithmetic mean of MINLP gap
- Time: shifted geometric mean of solving time in s with shift 5 s
- Solved: # solved instances
- 36 instances/cluster
- Initialize with optimal solution to compare strength of dual bounds.
- Use perspective cuts in validation MINLP.

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Resilience in Engineering

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Resilience of Technical Systems

A resilient technical system enables an operation even under disturbances or failure of system components to a pre-defined minimal functioning level.

Latin “resilire” – “rebound”, “return”

Resilience in Engineering

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Latin “resilire” – “rebound”, “return”

Paradigm Shift

What if? $\Rightarrow$ Whatever happens!

Resilience vs. Robust Optimization

Buffering Capacity

A design with connections x and pumps y has a buffering capacity of K if any failure of up to K pumps can be tolerated on reduced functioning level.

That is: system is robust against failure of up to K pumps (interdiction).

- Goal: find cost optimal design with buffering capacity K.
- Flow can still flow through pumps if they fail.
- For recourse strategy ignore energy consumption in case of failure.
- Model resilience for given fixed tree, since it is complex to model resilience with respect to connections x and pumps y using MINLP-techniques.

Buffering Capacity For Fixed Tree

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Set of pump failure scenarios:

$$\mathcal{Z} = \left\{ z \in \{0, \dots, M\}^{A \times \mathcal{C}} \,:\, \sum_{a \in A} \sum_{i \in \mathcal{C}} z_{a,i} \leq K \right\}$$

Theorem

A tree $T \in \mathcal{T}$ and pump purchase decision $y \in \{0, 1\}^{A \times \mathcal{C} \times [M]}$ has buffering capacity of K if and only if

$$\sum_{a \in \mathcal{P}_v^T} \sum_{i \in \mathcal{C}} \sum_{m=1+z_{a,i}}^{M} \overline{\Delta H}_i \Big(\tfrac{Q_a}{(m - z_{a,i})}, \tfrac{Q_a}{(m - z_{a,i})} \Big) \, y_{a,i}^m \geq H_v^{\min} \qquad (\star)$$

holds for each pressure zone $v \in V \setminus \{0\}$ and failure scenario $z \in \mathcal{Z}$.

Inclusion into Branch and Bound Algorithm

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- Dynamically separate ($\star$) since $\mathcal{Z}$ grows exponentially in K.
- Find a worst-case failure scenario $z \in \mathcal{Z}$ for given x and y.
- Solve for fixed pressure zone v

$$\min \quad \sum_{a \in \mathcal{P}_v^T} \sum_{i \in \mathcal{C}} \sum_{m=1+z_{a,i}}^{M} \overline{\Delta H}_i \left(\tfrac{Q_a}{(m-z_{a,i})}, \tfrac{Q_a}{(m-z_{a,i})} \right) y_{a,i}^m$$

$$\text{s.t.} \quad z \in \mathcal{Z} = \left\{ z \in \{0, \dots, M\}^{A \times \mathcal{C}} \;:\; \sum_{a \in A} \sum_{i \in \mathcal{C}} z_{a,i} \leq K \right\}.$$

- Structure similar to knapsack

Theorem

Theorem: Solvable by dynamic programming in $\mathcal{O}(N|\mathcal{C}|MK^2)$ steps.

Modified Branch and Bound Algorithm

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init $U = \infty$, $\mathcal{Z}' = \emptyset$ and $\mathcal{T} = \{T\}$ with $V_T = \{0, 1\}$ and $A_T = \{(0, 1)\}$;
while $\mathcal{T} \neq \emptyset$ **do**
  choose and remove a partial tree T from $\mathcal{T}$
  solve relaxation for T with Constraint $(\star)$ for $z \in \mathcal{Z}'$ and denote its optimal value O
  if $O < U$ *(else fathom node)*
    if *T spans G*
      solve exact problem for T by separating $(\star)$ and denote its optimal value O^*
      add separated scenarios to $\mathcal{Z}'$
      update $U = \min\{U, O^*\}$
    else
      form new trees from T and add to $\mathcal{T}$ (branching)
return U;

Optimal Solutions
Using specialized solution algorithm

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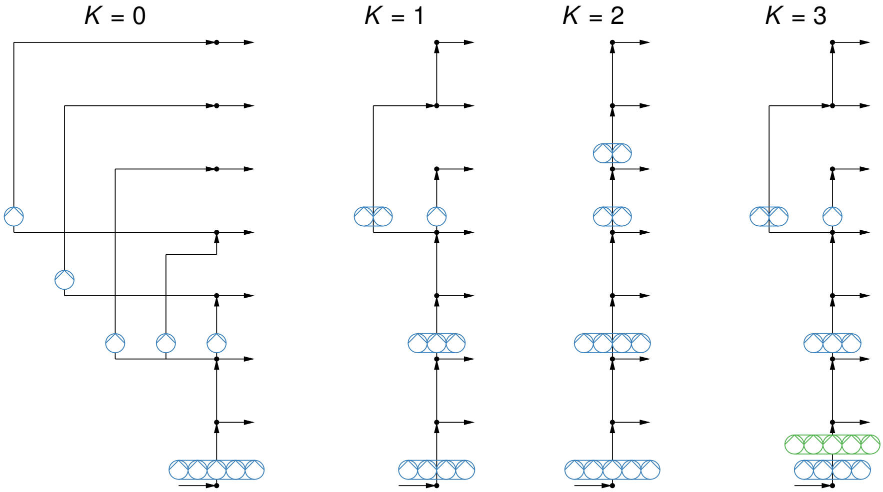

Cost Comparison

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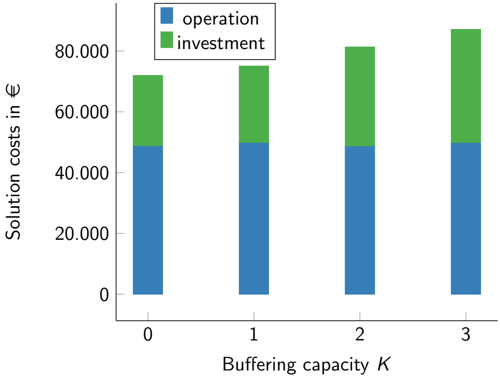

Higher resilience leads to greater overall costs, mainly due to investment costs.

Comparison of System Power

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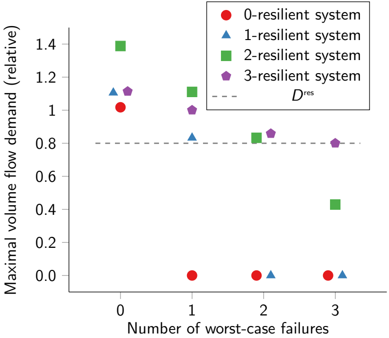
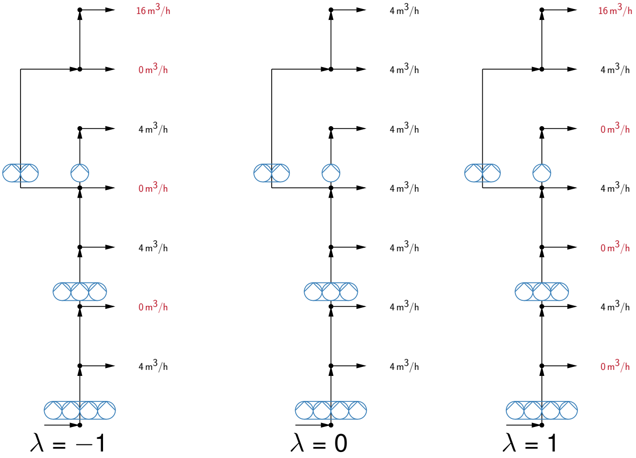
What is the maximal volume flow that can be transported after failures?

- Resilient solutions are oversized for standard operation.
- $K = 2$ solution has greatest reserves, thus resilience $\neq$ redundancy.



Robustness to demand shift

- Parameterize a change in demand using $\lambda \in [-1, 1]$.



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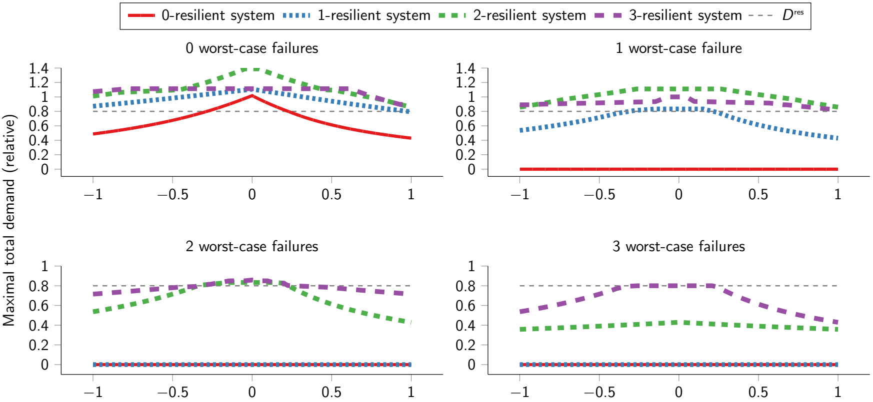

- Increased resilience leads to increased performance range and radius of performance.

Test Setting

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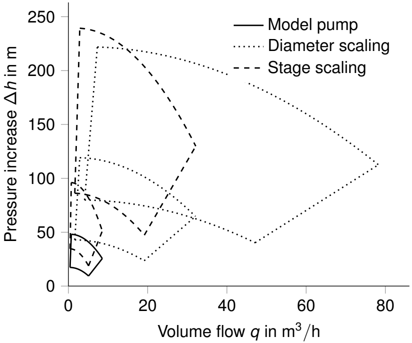
- Fixed parameters
  - 5 pump types, at most 3 parallel
  - fixed minimum pressure in each zone
  - fixed energy costs
- Variable parameters
  - building height [m]: $\{100, 150, 200\}$
  - flow demand [m^3/h]: $\{25, 30, 35\}$
  - number pressure zones: $\{4, 5, 6, 7, 8\}$
  - operating times [kh]: $\{10, 15, 20, 25\}$
- 180 instances



Computational Results
Branch and Bound

# zones		K				
		0	1	2	3	4
6	time	58.02	114.51	169.32	261.31	306.73
	solved	36	36	36	36	36
	$(\lvert\mathcal{Z}'\rvert/\lvert\mathcal{Z}\rvert) \cdot 100$	–	13.53	3.79	1.18	0.19
	enumerated sp. trees	1.00	1.00	0.96	0.80	0.65
7	time	412.62	847.73	1087.60	1252.01	1414.59
	solved	36	36	36	36	36
	$(\lvert\mathcal{Z}'\rvert/\lvert\mathcal{Z}\rvert) \cdot 100$	–	14.20	2.50	0.80	0.14
	enumerated sp. trees	1.00	0.99	0.95	0.76	0.61
8	time	3315.81	6388.67	6733.21	6570.97	6451.66
	solved	36	22	15	10	11
	$(\lvert\mathcal{Z}'\rvert/\lvert\mathcal{Z}\rvert) \cdot 100$	–	15.80	2.17	0.39	0.05
	enumerated sp. trees	1.00	0.98	0.89	0.61	0.48

Outline

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1 Water Distribution Networks – Model
2 Branch and Bound Solving Scheme
3 Resilience
4 Concluding Remarks

Conclusions

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Summary:

- Resilient water distribution networks can quite efficiently be computed using an adapted branch and bound algorithm.
- Key properties: tree shape, fixed flow, convexity of characteristic diagram
- treatment of component failures using a separation scheme

Future research:

- Try to adapt techniques to the case with cycles.
- Transfer of failure consideration to further applications.

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Thank you!